Errata for Technical Report "Quaderno 286"

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The following conjecture (mis-named *claim*) was made in [1, pp. 13-14].

Claim. Let $\mathcal{R} \Rrightarrow_\epsilon \mathcal{P} \neq \varnothing$, where $(\mathcal{C}, \mathcal{G}) \equiv \mathcal{R}$ is a minimal DD pair.

1. If $\mathcal{C}$ is in smf and either $\mathcal{C}_\epsilon = \varnothing$ or $\operatorname{con}(\mathcal{C} \setminus \mathcal{C}_\epsilon) \not\Rrightarrow_\epsilon \mathcal{P}$, then $\mathcal{G}$ is in smf.
2. If $\mathcal{G}$ is in smf and all the point encodings in $\mathcal{G}_P$ have the same value for the ϵ coordinate, then $\mathcal{C}$ is in smf.

These statements do not affect any other part of the cited paper and are not exploited in the implementation of the Parma Polyhedra Library described there. Nonetheless, both statements are false. To see this, we provide the following two counter-examples.

Counterexample to 1

Consider the polyhedron $\mathcal{P} \in \mathbb{P}_2$ where

$$\mathcal{P} = \left\{ (x, y)^{\mathrm{T}} \in \mathbb{R}^2 \;\middle|\; x > 0, y > 0, x < 3, y < 2 \right\}.$$

Then $\mathcal{R} = \operatorname{con}(\mathcal{C}) \in \mathbb{CP}_3$ is an ϵ-representation for $\mathcal{P}$ where

$$\mathcal{C} = \{x - \epsilon \geq 0, y - \epsilon \geq 0, -x - \epsilon \geq -3, -y - \epsilon \geq -2, \epsilon \geq 0\}.$$

Note that the constraint $\epsilon \leq 1$ is a consequence of the second and fourth of these constraints. Thus we have $\mathcal{C}_{\geq} = \mathcal{C}_\epsilon = \varnothing$. Then $\mathcal{R}$ (which may be visualized as a ridge or roof shape) is generated by $\mathcal{G} = (R, P)$ where $R = \varnothing$ and the point encodings $\mathcal{G}_P$ and closure point encodings $\mathcal{G}_C$ are as follows:

$$\begin{aligned}
\mathcal{G}_C &= \left\{(0,0,0)^{\mathrm{T}}, (0,2,0)^{\mathrm{T}}, (3,0,0)^{\mathrm{T}}, (3,2,0)^{\mathrm{T}}\right\}, \\
\mathcal{G}_P &= \left\{(1,1,1)^{\mathrm{T}}, (2,1,1)^{\mathrm{T}}\right\}.
\end{aligned}$$

Observe that $\mathcal{C}$ contains no ϵ-redundant constraints and hence is in smf, However, the point $\boldsymbol{p} = (1,1,1)^{\mathrm{T}}$ is in $\mathcal{G}_U$ so that (letting $\boldsymbol{p}' = (2,1,1)^{\mathrm{T}}$ in Definition 7) $\boldsymbol{p}$ is ϵ-redundant in $\mathcal{G}$. Therefore $\mathcal{G}$ is not in smf.

Counterexample to 2

Consider the polyhedron $\mathcal{P} = \{0\} \in \mathbb{CP}_1$. Then $\mathcal{R} = \operatorname{gen}(\mathcal{G}) \in \mathbb{CP}_2$ is an ϵ-representation for $\mathcal{P}$ where the generator system $\mathcal{G} = (R, P)$ is such that $R = \varnothing$

and P is the union of the closure point encodings $\mathcal{G}_C = \{(0,0)^{\mathrm{T}}\}$ and point encodings $\mathcal{G}_P = \{(0,1)^{\mathrm{T}}\}$. As $\mathcal{G}_P$ is a singleton, $\mathcal{G}$ is in smf for any constraint system defining $\mathcal{R}$. Moreover, the condition that all point encodings have the same value for ϵ is satisfied trivially. Consider the constraint system

$$\mathcal{C} = \{x \leq 0, x \geq 0, \epsilon \geq 0, x - \epsilon \geq -1\},$$

which is in minimal form. As the only point $(0,0)^{\mathrm{T}}$ in $\mathcal{G}_C$ does not saturate $c = (x - \epsilon \geq -1)$, c is ϵ-redundant (using the first condition in Definition 7). Therefore $\mathcal{C}$ is not in smf.

References

1. R. Bagnara, E. Ricci, E. Zaffanella, and P. M. Hill. Possibly not closed convex polyhedra and the Parma Polyhedra Library. Quaderno 286, Dipartimento di Matematica, Università di Parma, Italy, 2002. Available at `http://www.cs.unipr.it/Publications/`.