QUANTIFICATION IN FORMAL LOGIC AND NATURAL LANGUAGE

Formal Logic versus Linguistic Analysis- Formal Logic and Linguistics

My aim is an inquiry into the connections between logic and linguistics, that is to say into human mind and language.

This work takes for granted some version of the thesis that "things mental – that is minds- are emergent properties of brains."Such emergences are produced by principles that control the interactions between lower level events.

A key point is the kind of relationship between the elementary property of human language as "species property"or biological property, and the property of discrete infinity, which is exhibited in its purest form by natural numbers.Such properties might be considered as part of our biological endowment.

From the point of view of generative grammar, we know that diversity and complexity of human languages can be no more than appearance. They are variations of a single theme.Language structure must be invariant, except at the margins (Chomsky:1981,1993,1995,etc..).

As a consequence of this, we can state that each particular language can be derived from a uniform initial state under the boundary conditions set by experience.This is an explanation of the properties of languages at a deeper level.

It seems to be a universal characteristic of language that entities are regarded as divisible or indivisible, so things may be represented as quantifiable or unquantifiable.Indeed the categorization of things on this dimension is not fixed at an upper level.

Quantification is a notion of logic that has been in use in linguistics as well.To "quantify"- in its ordinary sense,- if there is one- means to point out a certain quantity of something.
The definition of the word in the Oxford English Dictionary is "determine quantity of, measure, express as quantity".The technical sense of the term is not far from this definition.

In order to understand why quantification is a necessary concept both in formal logic and in natural languages, we should consider that any common noun naming an object we may think of is associated with the totality of objects of the same kind.
Quantification is a means to quantify what proportion of that totality we have in mind.Let us consider some examples:
When we say "dogs bark" we refer to the totality of dogs.
In both " a dog is barking" and " the dog is barking" we refer to just one member of the set of all dogs.
In " the dogs are barking " we refer to all the members of the set of dogs within hearing distance,
In " some dogs do not bark " we point out that in the set of all dogs there are a number which do not bark.
"The", "a", "some", and the "zero article"are linguistic means of indicating among other things, what proportion of the set of all dogs the speaker has in mind.

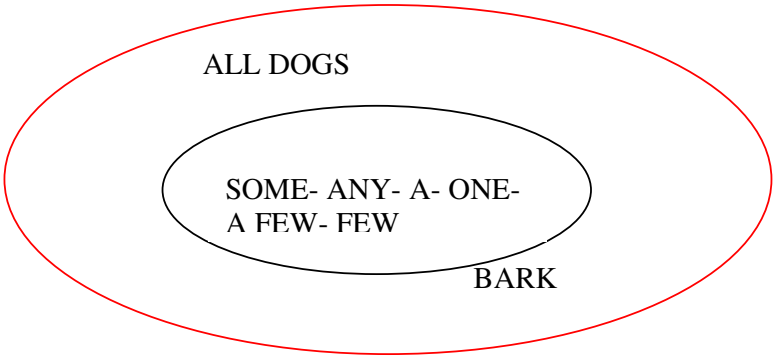

Fig.1

In logic, quantification is made by means of special operators called quantifiers.The role of the quantifiers can be explained in the following way: each proposition is seen as a relation between a number of arguments or nominal entities in linguistic terms.Thus, a distinction is made between individuals or objects which have properties or enter into certain relations and the properties they have or the relations they contract.

The arguments can be constant (e.g.proper names: John, Fido) or variables(x, y).The predicates are relations (e.g.verbs:love, walk).

Intransitive verbs translate into logic as one-place relations (e.g.leave, walk) and so do adjectives and common nouns(clever, boy, dog). In order to show that two elements, x and y, are in relation R with each other, we write R (x,y);if x has property P (a one-place relation) we write P(x).

If the arguments are constant, then a predicate and the respective arguments make up a proposition,e.g.:

Dog (Fido)- meaning " Fido is a dog ".
Clever(Fido)- meaning " Fido is clever ".
Love(Fido, John)-"Fido loves John ".
Leave (John)- "John is leaving".
Walk(John, Fido)- "John is going to walk Fido".

The sequences above are propositions;they make sense from a logical point of view – they can be interpreted, i.e.they can be assigned one of the two " meanings" with which logic operates:True (T) or False (F).

If, on the other hand,one or more arguments are variables, the predicate and the variables alone do not form a proposition, i.e. a sequence that can be assigned one of the values T or F; they form a propositional function, which is a sequence that can only be interpreted if additional information is given regarding the variables involved.

More precisely, variables cover a certain domain (a set of objects), and in order to be able to say whether a sequence containing variables is true or false, we have to know what the spread of the variable(s) is, the portion occupied in the set of variables of certain type. This can be specified by means of quantifiers.

A UNIVERSAL SEQUENCE IS TRUE IF AND ONLY IF ALL CASES ARE TRUE
AN EXISTENTIAL SEQUENCE IS TRUE IF AND ONLY IF AT LEAST ONE CASE IS TRUE

Fig. 2

There are two quantifiers in formal logic: the 'Universal quantifier'$\forall$ – which shows that the whole set is covered, and the 'Existential quantifier' $\exists$ – which shows that at least one member of the set is referred to.

The quantifiers are said to 'bind' the variables. A sequence- in which the variables bound by quantifiers can be interpreted as either T or F - is a proposition.

e.g.	$\forall$ (x) Dog(x) – Clever(x) (implies)	('Dogs are clever' 'All dogs are clever')
	$\forall$ (x) Dog(x) – Bark(x) (implies)	('Dogs bark' 'All dogs bark')
	$\exists$ (x) Dog(x) and Not Bark(x)	('Some dogs don't bark' 'There are dogs which don't bark')
	$\forall$(x), $\forall$(y) Dog(x) and Boy(y) - Love(x,y)	('Dogs love boys'; 'All dogs love boys')

$\exists(x),\forall(y)$ Dog(x) and Boy(y) and Not Love(x,y) (‘Some dogs do not love boys’; ‘There are dogs which do not love boys’).

Formal logic – an artificial language – operates with two quantifiers only. In natural languages like English we have quite a number of linguistic devices, which, if we translate English into formal logic, will be reduced to the two mentioned above.

Thus, likely equivalents of the Universal quantifier are:”each”, “all”, “both”, “every”, “any”, and of the Existential quantifier:”any”, “no”, “some”.

$\forall$ Dogs are human beings which perform the action of barking

$\exists$ Some dogs- perhaps ten, three, at least one member of the group – do not perform it

Since we study language from a linguistic, not a mathematical or logic point of view, we are not interested in reducing natural language expressions to a restricted number of logical notions, but rather to use the notions of logical in order to better understand the differences in meaning of the variuos linguistic expressions.(Chomsky: ;Haegeman:1994).

By analogy with the meaning at the basis of the logical notion of quantification, expressions in natural language which show quantity have also been called quantifiers: many, several, some hundreds/ hoards of, a glass/ yard/ kilo of, etc. Many of these ‘quantifiers’ fill a certain slot in the noun phrase, so that their grouping together is justified both according to their meaning and to their distribution.

Quantifiers limit the scope of reference of things. They provide information about “how much entity”, or “how many “ entities, not about “which” entities (they are provided by ‘determiners’). They indicate a quantity somewhere between ‘all’ and ‘ none’. We can represent the qualitative aspect of this concept on the diagram below:

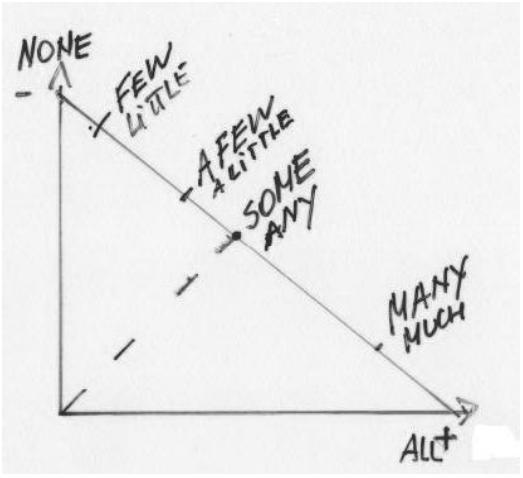

Fig. 3

Linguistic approaches to quantifiers consider such linguistic entities as belonging to different domains.If we have a look at the most updated grammars of the English language, we can find quantifiers among the logic- syntactic domains of articles, determiners, partitives , nouns,or numerals.(J.Eastwood:1999, Oxford Practice Grammar; Swan and Walker: 2001, The Good Grammar Book).

The parameters of the University of Cambridge for the acquisition of English as a second language locate quantifiers as discrete entities in the following linguistic framework aimed at testing basic skills:
(It applies principles and researches carried out by ALTE.The association of Language Testers in Europe has developed a general framework of linguistic skills, which shows what language learners should be able to do typically do at each level aligned to the Council of Europe Common European Framework).

NOUNS
Singular and plural- abstract- compound-
Complex noun phrases
Genitive- double genitive
PRONOUNS
Personal- Reflexive and emphatic-Impersonal-
Demonstrative:this, that,these, those
Quantitative: one, something,everybody, ...
Indefinite:some, any, something, one.....
Relative: who, which,that, whom,whose
DETERMINERS
A+countable nouns
The+ countable/uncountable nouns
ADJECTIVES
Colour,shape,size, nationality,quality
Predicative and attributive
Cardinal and ordinal numbers
Possessive
Demonstrative
Quantitative:some, any, many, much, a few, a lot of, all, other, every, etc...
Comparative and superlative forms...(.as....as; not .....enough to, too...)
ADVERBS
Manner- frequency- degree- definite time- indefinite time-
Place- direction- sequence-
Comparative and superlative forms

Another kind of distinction is the one between 'Inexact' and 'Exact' quantifiers.
Inexact quantifiers include " many, much, a lot of several, some, any, a few, little,fewer, fewest, less, least, more, most".

A potential problem lies in the way these quantifiers can co- occur with Referrers, for example, " Their many supporters" is acceptable,whereas "their much help" is not).(Wilkins:1983).

" Many", "several", "a few", "fewer",and "fewest" are only used with 'Count Nouns'. "Much," "little", "a little" are used with 'mass nouns'.
"Some", "any"," a lot of" , "least", "less" may be used with both 'mass' and 'count' nouns.
"Some" and "any" have a range of uses thay could justify their inclusion in the class of articles.
The nature of quantifiers is shown in sentences before 'mass nouns' and plural 'count nouns', e.g.:
" There are some interesting books in the library".

The difference between a ‘mass noun’ and a ‘count noun’ is based on meaning; the former represents things occurring as ‘indifferentiated whole’(for example, materials, substances like wood, oil, wine; or abstract entities like intelligence, love).The latter represents things occurring in the form of ‘discrete entities’ (e.g. dog, book, table, or names of parts of a whole, bit, slice .....).

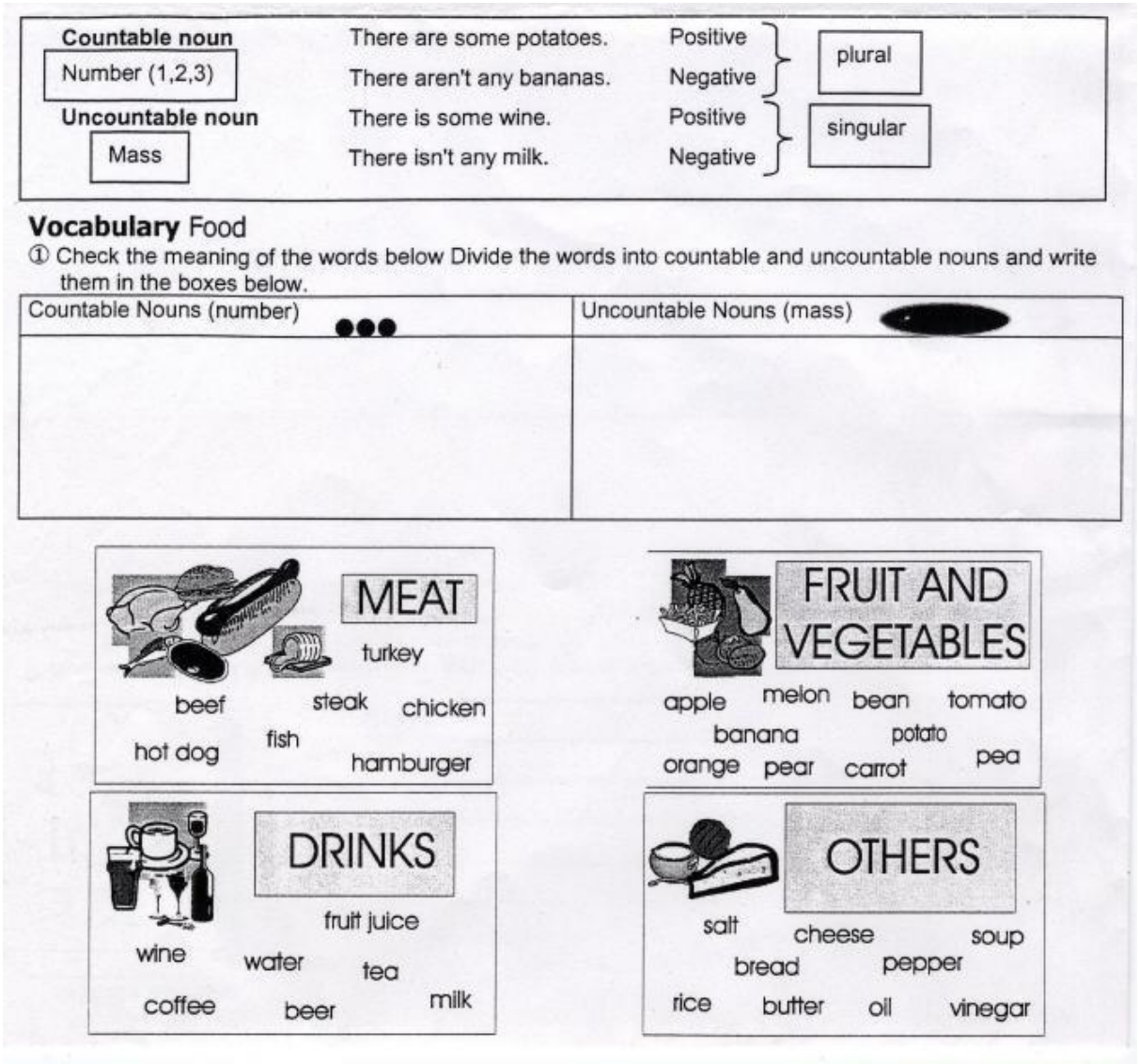

Fig.4

The following names are used as ‘mass’ nouns in English, not in Italian-: furniture, luggage, advice, machinery, information- though it would be interesting to consider whether there are any contexts in which it might be acceptable to use them as ‘count nouns’.

In fact, we should classify ‘the uses’ of nouns into ‘count’ or ‘mass’, if we consider that many nouns can be used both as count’ nouns and ‘mass’ nouns (e.g. chocolate, cheese, experience, war, business, work....)

In order to indicate the concept of 'exact quantity' Natural Languages use'exact quantifiers'that is to say 'numerals'such as one, ten twenty, thirty, and so on.
The meaning of numerals is based on the concept of mathematical number, limited effectively by a single distinguishing feature, which isolates the given number from a serie of numbers.

In many languages we find inherent in numerals two forms corresponding to their functional groups- cardinal, ordinal- but in other languages there are three forms of numerals.In Chinese, for example, we should consider three different forms of numerals, absolute, correlated and ordinal.

Numeric quantity is also expressed by the use of plural numerals, such as dozens, hundreds, millions of....

Partitives should be considered as 'complex' quantifiers, if we consider the structure of the relationship between two nouns connected by 'of ', for example: "a lot of students have arrived"; "a number of " boys have played the game (it is the name students/ boys which determines the agreement of the verb).

"A few" and "a little" simply indicate a small quantity, whereas "few" and "little" often have a negative connotation, for example in the case that the quantity is insufficient, less than expected or hoped, and so on.

The recent literature on 'indefinites' and 'quantifiers' contains a number of stimulating hyphothesis that we are going to discuss in a further conference.Here, the linguistic 'melting pot' of quantifiers has been analyzed from a specific viewpoint.Both 'exact' and 'inexact' quantifiers belong to existential quantifiers, which refer to universal quantifiers.

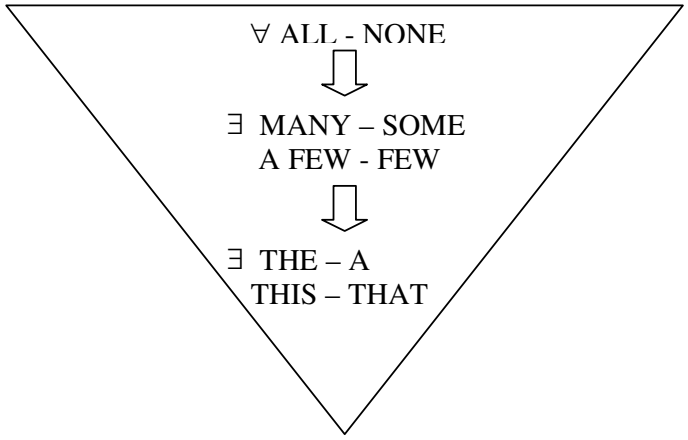

Fig. 5

Our aim was to demonstrate that Natural languages like English have words which serve to identify the separate numbers of plurality in the singular form and whose meanings express that form which characterizes all members of that number.

We have considered that the concept of quantity is expressed by an exclusively generalized type of sign meaning .

Formal logic states that such signs and meanings correspond to concepts embedded in the human mind.

A further reflection on the relationship we have investigated could be put forward in the following way, adopting concepts expressed by Chomsky last March: "Selbst der Grossteil der Naturwissenschaften ist deskriptiv.....Sie stellen sich komplizierte Wege dar, um etwas zu sagen, was man auch in einfachen Worten sagen und damit fur die Leute verstandlich machen kann. (Chomsky: 24.03.2004,'Wiener Zeitung').

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